CHAPTER 5

Applications of Newton's Laws

1* • Various objects lie on the floor of a truck moving along a horizontal road. If the truck accelerates, what force acts on the objects to cause them to accelerate?
Force of friction between the objects and the floor of the truck.

2 • Any object resting on the floor of a truck will slide if the truck's acceleration is too great. How does the critical acceleration at which a light object slips compare with that at which a much heavier object slips?
They are the same.

3 • True or false: (*a*) The force of static friction always equals $\mu_s F_n$. (*b*) The force of friction always opposes the motion of an object. (*c*) The force of friction always opposes sliding. (*d*) The force of kinetic friction always equals $\mu_k F_n$.
(*a*) False (*b*) True (*c*) True (*d*) True

4 • A block of mass *m* rests on a plane inclined at an angle θ with the horizontal. It follows that the coefficient of static friction between the block and the plane is (*a*) $\mu_s \geq 1$. (*b*) $\mu_s = \tan\theta$. (*c*) $\mu_s \leq \tan\theta$. (*d*) $\mu_s \geq \tan\theta$.
(*d*)

5* • A block of mass *m* is at rest on a plane inclined at angle of 30° with the horizontal, as in Figure 5-38. Which of the following statements about the force of static friction is true? (*a*) $f_s > mg$ (*b*) $f_s > mg\cos 30^\circ$
(*c*) $f_s = mg\cos 30^\circ$ (*d*) $f_s = mg\sin 30^\circ$ (*e*) None of these statements is true.
(*d*) f_s must equal in magnitude the component of the weight along the plane.

6 • A block of mass *m* slides at constant speed down a plane inclined at an angle θ with the horizontal. It follows that (*a*) $\mu_k = mg\sin\theta$. (*b*) $\mu_k = \tan\theta$. (*c*) $\mu_k = 1 - \cos\theta$. (*d*) $\mu_k = \cos\theta - \sin\theta$.
(*a*) Acceleration = 0, therefore $f_k = mg\sin\theta$. With $F_n = mg\cos\theta$, it follows that $\mu_k = \tan\theta$

7 • A block of wood is pulled by a horizontal string across a horizontal surface at constant velocity with a force of 20 N. The coefficient of kinetic friction between the surfaces is 0.3. The force of friction is (*a*) impossible to determine without knowing the mass of the block. (*b*) impossible to determine without knowing the speed of the block. (*c*) 0.3 N. (*d*) 6 N. (*e*) 20 N.
(*e*) The net force is zero.

8 · A 20-N block rests on a horizontal surface. The coefficients of static and kinetic friction between the surface and the block are $\mu_s = 0.8$ and $\mu_k = 0.6$. A horizontal string is attached to the block and a constant tension T is maintained in the string. What is the force of friction acting on the block if (*a*) $T = 15$ N, or (*b*) $T = 20$ N.

(*a*) If $\mu_s mg > 15$, then $f = f_s = 15$ N — $0.8 \times (20\text{ N}) = 16$ N; $f = f_s = 15$ N

(*b*) $T > f_{s,max}$; $f = f_k = \mu_k mg$ — $f = f_k = 0.6 \times (20\text{ N}) = 12$ N

9* · A block of mass m is pulled at constant velocity across a horizontal surface by a string as in Figure 5-39. The magnitude of the frictional force is (*a*) $\mu_k mg$. (*b*) $T\cos\theta$. (*c*) $\mu_k(T - mg)$. (*d*) $\mu_k T\sin\theta$. (*e*) $\mu_k(mg + T\sin\theta)$.

(*b*) The net force is zero.

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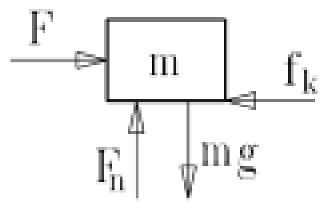
10 · A tired worker pushes with a force of 500 N on a 100-kg crate resting on a thick pile carpet. The coefficients of static and kinetic friction are 0.6 and 0.4, respectively. Find the frictional force exerted by the surface.

1. Draw the free-body diagram



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $F_n - (100 \times 9.81)\text{ N} = 0$; $F_n = 981$ N
3. $f_{s,max} = \mu_s F_n$ — $f_{s,max} = 589\text{ N} > 500$ N
4. Since 500 N < $f_{s,max}$ the box does not move — $F = f_s = 500$ N

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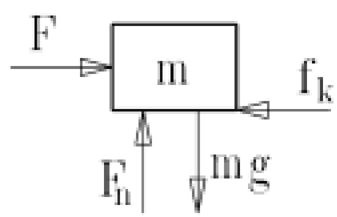
11 · A box weighing 600 N is pushed along a horizontal floor at constant velocity with a force of 250 N parallel to the floor. What is the coefficient of kinetic friction between the box and the floor?

Draw the free-body diagram.



1. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$; $a_x = a_y = 0$ — $250\text{ N} = f_k$; $F_n = 600$ N
2. $f_k = \mu_k F_n$; solve for μ_k — $\mu_k = (250/600) = 0.417$

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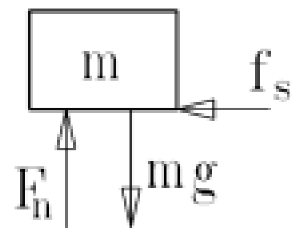
12 · The coefficient of static friction between the tires of a car and a horizontal road is $\mu_s = 0.6$. If the net force on the car is the force of static friction exerted by the road, (*a*) what is the maximum acceleration of the car when it is braked? (*b*) What is the least distance in which the car can stop if it is initially traveling at 30 m/s?

Draw the free-body diagram.



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(a) 1. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$; $a_y = 0$ — $f_{s,\max} = ma_{max}$; $F_n = mg$

2. Use $f_{s,\max} = \mu_s F_n$ and solve for $a_{\max}$ — $a_{\max} = \mu_s g = (0.6 \times 9.81)$ m/s^2 = 5.89 m/s^2.

(b) Use $v^2 = v_o^2 + 2as$; solve for s with $v = 0$ — $s = [30^2/(2 \times 5.89)]$ m = 76.5 m

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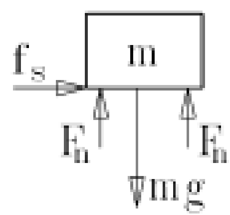
13* · The force that accelerates a car along a flat road is the frictional force exerted by the road on the car's tires. (a) Explain why the acceleration can be greater when the wheels do not spin. (b) If a car is to accelerate from 0 to 90 km/h in 12 s at constant acceleration, what is the minimum coefficient of friction needed between the road and tires? Assume that half the weight of the car is supported by the drive wheels.

(a) $\mu_s > \mu_k$; therefore f is greater if the wheels do not spin.



(b) 1. Draw the free-body diagram; the normal force on each pair of wheels is $\frac{1}{2}mg$.

2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $f_s = ma = \mu_s F_n$; $F_n = \frac{1}{2}mg$

3. Solve for a — $a = \frac{1}{2}\mu_s g = (25\text{ m/s}^2)/(12\text{ s}) = 2.08\text{ m/s}^2$

4. Find μ_s — $\mu_s = (2 \times 2.08/9.81) = 0.425$

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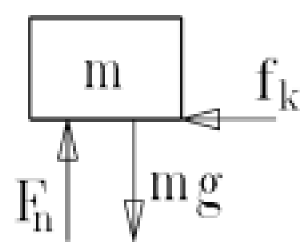
14 · On the current tour of the rock band Dead Wait, the show opens with a dark stage. Suddenly there is the sound of a large automobile accident. Lead singer Sharika comes sliding to the front of the stage on her knees. Her initial speed is 3 m/s. After sliding 2 m, she comes to rest in a dry ice fog as flash pots explode on either side. What is the coefficient of kinetic friction between Sharika and the stage?

Draw the free-body diagram.



1. Use $v^2 = v_0^2 + 2as$; solve for a — $a = (9/4)$ m/s^2

2. $f_k = ma = \mu_k F_n$; $F_n = mg$ — $\mu_k = a/g = [9/(4 \times 9.81)] = 0.23$

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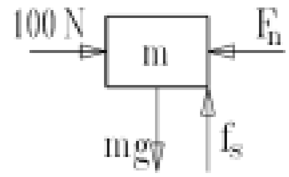
15 · A 5-kg block is held at rest against a vertical wall by a horizontal force of 100 N. (*a*) What is the frictional force exerted by the wall on the block? (*b*) What is the minimum horizontal force needed to prevent the block from falling if the coefficient of friction between the wall and the block is $\mu_s = 0.40$?

(*a*) 1. Draw the free-body diagram



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $F_n = 100$ N; $f_s = mg = 49.05$ N

(*b*) $f_s = \mu_s F_n$; solve for F_n — $F_n = (49.05\text{ N})/0.4 = 123$ N

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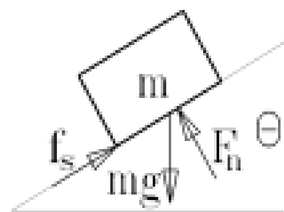
16 · On a snowy day with the temperature near the freezing point, the coefficient of static friction between a car's tires and an icy road is 0.08. What is the maximum incline that this four-wheel-drive vehicle can climb with zero acceleration?

1. Draw the free-body diagram



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $f_s - mg\sin\theta = 0$; $f_s = mg\sin\theta$

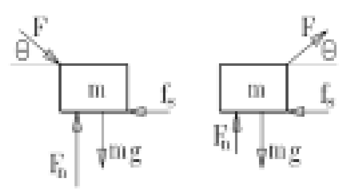
$F_n - mg\cos\theta = 0$; $F_n = mg\sin\theta$

3. $f_s = \mu_s F_n$ — $mg\sin\theta = \mu_s mg\cos\theta$

4. Solve for μ_s and find θ — $\mu_s = \tan\theta = 0.08$; $\theta = \tan^{-1}(0.08) = 4.57^\circ$

17* · A 50-kg box that is resting on a level floor must be moved. The coefficient of static friction between the box and the floor is 0.6. One way to move the box is to push down on it at an angle θ with the horizontal. Another method is to pull up on the box at an angle θ with the horizontal. (*a*) Explain why one method is better than the other. (*b*) Calculate the force necessary to move the box by each method if $\theta = 30^\circ$ and compare the answers with the result when $\theta = 0^\circ$.

The free-body diagram for both cases, $\theta > 0$ and $\theta < 0$, is shown.



(*a*) $\theta > 0$ is preferable; it reduces F_n and therefore f_s.

(*b*) 1. Use $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ to determine F_n — $F \sin\theta + F_n - mg = 0$. $F_n = mg - F\sin\theta$

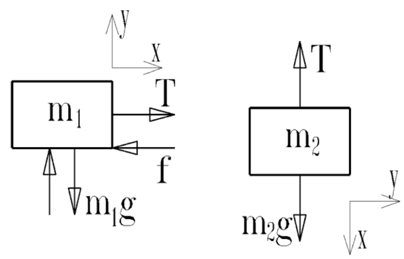
2. $f_{s,max} = \mu_s F_n$ — $f_{s,max} = \mu_s(mg - F\sin\theta)$

3. To move the box, $F_x = F\cos\theta \geq f_{s,max}$ — $F = \mu_s(mg - F\sin\theta)/\cos\theta$; $F = \dfrac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$

4. Find F for $m = 50$ kg, $\mu_s = 0.6$, and $\theta = 30^\circ$, $\theta = -30^\circ$, and $\theta = 0^\circ$ — $F(30^\circ) = 252$ N, $F(-30^\circ) = 520$ N, $F(0^\circ) = 294$ N

18 · A 3-kg box resting on a horizontal shelf is attached to a 2-kg box by a light string as in Figure 5-40. (*a*) What is the minimum coefficient of static friction such that the objects remain at rest? (*b*) If the coefficient of static friction is less than that found in part (*a*), and the coefficient of kinetic friction between the box and the shelf is 0.3, find the time for the 2-kg mass to fall 2 m to the floor if the system starts from rest.

1. Draw a free-body diagram for each object. In the absence of friction, m_1 will move to the right, m_2 will move down. The friction force is indicated by $\boldsymbol{f}$ without subscript; it is $\boldsymbol{f}_s$ for (*a*), $\boldsymbol{f}_k$ for (*b*).



$F_n - m_1g = 0$; $T - f = 0$; $m_2g - T = 0$; $f = f_s = T = m_2g$

(*a*) 1. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ for each mass. Note that $\boldsymbol{a} = 0$ — $F_n = m_1g$

2. $f_s = f_{s,max} = \mu_s F_n$; solve for μ_s — $m_2g = \mu_s m_1 g$; $\mu_s = m_2/m_1 = 2/3 = 0.667$

(*b*) If $\mu_s < 0.667$, the system will accelerate. — $F_n = m_1g$; $T - f_k = m_1a$; $m_2g - T = m_2a$; $f_k = \mu_k m_1 g$

1. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$; $a_y = 0$; $a = a_x$ — $a = (m_2 - \mu_k m_1)g/(m_1 + m_2)$

2. Solve for a — $a = 2.16$ m/s^2

3. Find a for $m_1 = 3$ kg, $m_2 = 2$ kg, $\mu_k = 0.3$ — $t = (2s/a)^{1/2} = (2 \times 2/2.16)^{1/2}$ s $= 1.36$ s

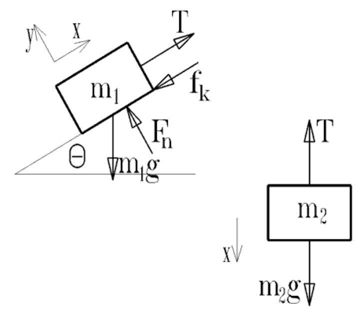
4. Use $s = \frac{1}{2}at^2$; solve for and find t

19 ·· A block on a horizontal plane is given an initial velocity v. It comes to rest after a displacement d. The coefficient of kinetic friction between the block and the plane is given by (*a*) $\mu_k = v^2d/2g$. (*b*) $\mu_k = v^2/2dg$. (*c*) $\mu_k = v^2g/d^2$. (*d*) none of the above.

(*b*) $v^2 = 2ad$; $a = f_k/m = \mu_k mg/m = \mu_k g$; solve for μ_k.

20 ·· A block of mass $m_1 = 250$ g is at rest on a plane that makes an angle $\theta = 30^\circ$ above the horizontal (Figure 5-41). The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.100$. The block is attached to a second block of mass $m_2 = 200$ g that hangs freely by a string that passes over a frictionless and massless pulley. When the second block has fallen 30.0 cm, its speed is (*a*) 83 cm/s. (*b*) 48 cm/s. (*c*) 160 cm/s. (*d*) 59 cm/s. (*e*) 72 cm/s.

1. Draw free-body diagrams for each object.
Note that both objects have the same acceleration.



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ to each object and use $f_k = \mu_k F_n$

$T - m_1 g \sin\theta - f_k = m_1 a$; $m_1 g \cos\theta = F_n$; $m_2 g - T = m_2 a$

$T = m_2 g - m_2 a$; $f_k = \mu_k m_1 g \cos\theta$

3. Solve for a

$$a = \frac{[m_2 - m_1(\sin\theta + \mu_k \cos\theta)]g}{m_1 + m_2}\ 0;\quad a = 1.16\ \mathrm{m/s^2}$$

4. Find $v = (2ad)^{1/2}$

$v = (2 \times 1.16 \times 0.3)^{1/2} = 0.83$ m; (*a*) is correct.

21* ·· Returning to Figure 5-41, this time $m_1 = 4$ kg. The coefficient of static friction between the block and the incline is 0.4. (*a*) Find the range of possible values for m_2 for which the system will be in static equilibrium. (*b*) What is the frictional force on the 4-kg block if $m_1 = 1$ kg?

(*a*) 1. Use the result of Problem 5-20; set $a = 0$. Note that f_s may point up or down the plane.

$0 = m_2 - m_1(\sin\theta \pm \mu_s \cos\theta)$

2. Solve for m_2 with $m_1 = 4$ kg, $\mu_s = 0.4$

$m_2 = 3.39$ kg, 0.614 kg.

$m_{2,\mathrm{max}} = 3.39$ kg, $m_{2,\mathrm{min}} = 0.614$ kg

(*b*) 1. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$; set $\boldsymbol{a} = 0$

$m_2 g + f_s - m_1 g \sin\theta = 0$

2. Solve for and find f_s

$f_s = [(4.0 \times 0.5 - 1.0) \times 9.81]$ N $= 9.81$ N

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22 ·· Returning once again to Figure 5-41, this time $m_1 = 4$ kg, $m_2 = 5$ kg, and the coefficient of kinetic friction between the inclined plane and the 4-kg block is $\mu_k = 0.24$. Find the acceleration of the masses and the tension in the cord.

1. Use the result of Problem 5-20; substitute numerical values. $a = 2.36\ \mathrm{m/s^2}$

2. Use $T = m_2 g - m_2 a$ and Problem 5-20 to obtain an

$$T = \frac{m_1 m_2 g(1 + \sin\theta + \mu_s \cos\theta)}{m_1 + m_2}\ 0;\ T = 37.2\ \mathrm{N}$$

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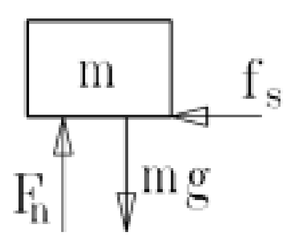
23 ·· The coefficient of static friction between the bed of a truck and a box resting on it is 0.30. The truck is traveling at 80 km/h along a horizontal road. What is the least distance in which the truck can stop if the box is not to slide?

1. Draw the free-body diagram.



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ to find a_{max}

$f_{s,\mathrm{max}} = \mu_s mg = ma_{\mathrm{max}}$; $a_{\mathrm{ax}} = \mu_s g$

3. Use $v^2 = v_0^2 + 2ax$; solve for $x = d_{\mathrm{min}}$

$d_{\mathrm{min}} = (v_0^2/2a_{\mathrm{max}})^{1/2} = (v_0^2/2\mu_s g)^{1/2}$; $d_{\mathrm{min}} = 83.9$ m

24 ·· A 4.5-kg mass is given an initial velocity of 14 m/s up an incline that makes an angle of 37° with the horizontal. When its displacement is 8.0 m, its upward velocity has diminished to 5.2 m/s. Find (*a*) the coefficient of kinetic friction between the mass and the plane, (*b*) the displacement of the mass from its starting point at the time when it momentarily comes to rest, and (*c*) the speed of the block when it again reaches its initial position.

Draw the free-body diagram

(*a*) 1. Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$ — $F_n = mg\cos\theta$; $ma_x = -mg\cos\theta - f_k$

2. Replace $f_k = \mu_k F_n$ and solve for a_x — $a_x = -(\sin\theta + \mu_k\cos\theta)g$

3. Use $a_x = (v^2 - v_0^2)/2s$ and solve for μ_k — $\mu_k = (v_0^2 - v^2)/(2gs\cos\theta) - \tan\theta$; $\mu_k = 0.594$

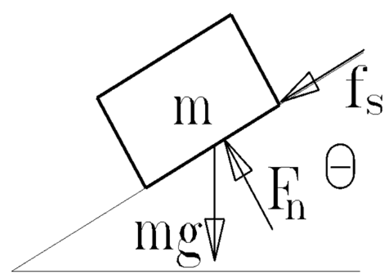
(*b*) Set $v^2 = 0$ and solve for *s*. — $s = v_0^2/2g(\sin\theta + \mu_k\cos\theta)$; $s = 9.28$ m

(*c*) 1. Note that now f_k points upward; write a_x — $a_x = (\mu_k\cos\theta - \sin\theta)g$

2. $v = (2as)^{1/2}$; note that *s* and *a* are negative and solve for and evaluate *v* — $v = v_0\sqrt{\dfrac{\sin\theta - \mu_k\cos\theta}{\sin\theta + \mu_k\cos\theta}}$ 0; $v = 4.82$ m/s

25* ·· An automobile is going up a grade of 15° at a speed of 30 m/s. The coefficient of static friction between the tires and the road is 0.7. (*a*) What minimum distance does it take to stop the car? (*b*) What minimum distance would it take if the car were going down the grade?

The free-body diagram is shown for part (*a*).



For part (*b*), $\boldsymbol{f}_s$ points upward along the plane.

(*a*) We can use the result of Problem 5-24(*a*) and (*b*), replacing μ_k by μ_s — $s = v_0^2/2g(\sin\theta + \mu_s\cos\theta)$; $s = 49.1$ m

(*b*) Replace $\sin\theta$ by $-\sin\theta$ — $s = v_0^2/2g(\mu_s\cos\theta - \sin\theta)$; $s = 110$ m

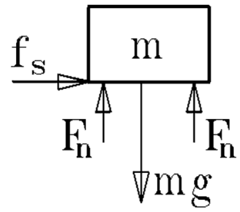
26 ·· A block of mass *m* slides with initial speed v_0 on a horizontal surface. If the coefficient of kinetic friction between the block and the surface is μ_k, find the distance *d* that the block moves before coming to rest.

The problem is essentially identical to Problem 5-14, with $s = d$ the unknown. $a = -\mu_k g$ and $d = -v_0^2/2a = v_0^2/2\mu_k g$.

27 ·· A rear-wheel-drive car supports 40% of its weight on its two drive wheels and has a coefficient of static friction of 0.7. (*a*) What is the vehicle's maximum acceleration? (*b*) What is the shortest possible time in which this car can achieve a speed of 100 km/h? (Assume that the engine has unlimited power.)

Draw the free-body diagram.

$f_{s,max} = 0.4\mu_s mg$.



(*a*) Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $0.4\mu_s mg = ma$; $a = 0.4\mu_s g$; $a = 2.75\ \mathrm{m/s^2}$

(*b*) $v = at$; solve for and find t — $t = v/a$; $t = (27.8/2.75)$ s = 10.1 s

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28 ·· Lou bets an innocent stranger that he can place a 2-kg block against the side of a cart, as in Figure 5-42, and that the block will not fall to the ground, even though Lou will use no hooks, ropes, fasteners, magnets, glue, or adhesives of any kind. When the stranger accepts the bet, Lou begins to push the cart in the direction shown. The coefficient of static friction between the block and the cart is 0.6. (*a*) Find the minimum acceleration for which Lou will win the bet. (*b*) What is the magnitude of the frictional force in this case? (*c*) Find the force of friction on the block if *a* is twice the minimum needed for the block not to fall. (*d*) Show that, for a block of any mass, the block will not fall if the acceleration is $a \geq g/\mu_s$, where μ_s is the coefficient of static friction.

(a) 1. The normal force acting on the block is the force exerted by the cart.

2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $F_n = F = ma$

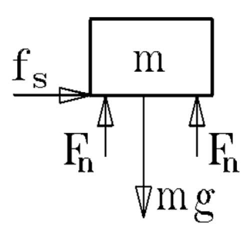
(*b*) $f_s = f_{s,max}$ — $f_{s,max} = \mu_s ma = mg$; $a_{min} = g/\mu_s = 16.4\ \mathrm{m/s^2}$

(*c*) f_s is again mg — $f_s = mg = 19.6$ N

(*d*) Since g/μ_s is a_{min}, block will not fall if $a \geq g/\mu_s$ — $f_s = 19.6$ N

29* ·· Two blocks attached by a string slide down a 20° incline. The lower block has a mass of $m_1 = 0.25$ kg and a coefficient of kinetic friction $\mu_k = 0.2$. For the upper block, $m_2 = 0.8$ kg and $\mu_k = 0.3$. Find (*a*) the acceleration of the blocks and (*b*) the tension in the string.

(*a*) 1. Draw the free-body diagrams for each block. Since the coefficient of friction for the lower block is the smaller, the string will be under tension.



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ to each block — $T + f_{1k} - m_1 g\sin\theta = m_1 a$ $-T + f_{2k} - m_2 g\sin\theta = m_2 a$; $F_{1n} - m_1 g\cos\theta = 0$ $F_{2n} - m_2 g\cos\theta = 0$

3. Add the first pair of equations; use $f_k = \mu_k F_n$ — $(m_1\mu_{1k} + m_2\mu_{2k})g\cos\theta - (m_1 + m_2)g\sin\theta = (m_1 + m_2)a$

4. Solve for a

$$a = \frac{(m_1\mu_{1k} + m_2\mu_{2k})\cos\theta - (m_1 + m_2)\sin\theta}{m_1 + m_2}g$$

5. Solve for T

$$T = \frac{m_1 m_2(\mu_{2k} - \mu_{1k})g\cos\theta}{m_1 + m_2}$$

6. Substitute numerical values for the masses, friction coefficients, and θ to find a and T. — $a = -0.809$ m/s^2 (i.e., down the plane); $T = 0.176$ N

30 ·· Two blocks attached by a string are at rest on an inclined surface. The lower block has a mass of $m_1 = 0.2$ kg and a coefficient of static friction $\mu_s = 0.4$. The upper block has a mass $m_2 = 0.1$ kg and $\mu_s = 0.6$. (*a*) At what angle θ_c do the blocks begin to slide? (*b*) What is the tension in the string just before sliding begins?

(a) 1. Referring to Problem 5-29, replace μ_k by μ_s and set $a = 0$. — $(m_1\mu_{1s} + m_2\mu_{2s})\cos\theta - (m_1 + m_2)\sin\theta = 0$

2. Solve for and evaluate $\theta = \theta_c$ — $\theta_c = \tan^{-1}[(m_1\mu_{1s} + m_2\mu_{2s})/(m_1 + m_2)$; $\theta_c = 25^\circ$

(b) 1. Since $\tan^{-1}(0.4) = 21.8^\circ$, lower block would slide if $T = 0$. Set $a = 0$ and solve for T — $m_1 g\sin\theta - \mu_{1s}m_1 g\cos\theta - T = 0$; $T = m_1 g\sin\theta - \mu_{1s}m_1 g\cos\theta$

2. Evaluate T — $T = 0.118$ N

31 ·· Two blocks connected by a massless, rigid rod slide on a surface inclined at an angle of 20°. The lower block has a mass $m_1 = 1.2$ kg, and the upper block's mass is $m_2 = 0.75$ kg. (*a*) If the coefficients of kinetic friction are $\mu_k = 0.3$ for the lower block and $\mu_k = 0.2$ for the upper block, what is the acceleration of the blocks? (*b*) Determine the force transmitted by the rod.

(*a*), (*b*) We can use the results of Problem 5-29 and — $a = -0.944$ m/s^2 (downward acceleration); $T = -0.425$ N (rod under compression)

32 ·· A block of mass m rests on a horizontal surface (Figure 5-43). The box is pulled by a massless rope with a force $\boldsymbol{F}$ at an angle θ . The coefficient of static friction is 0.6. The minimum value of the force needed to move the block depends on the angle θ. (*a*) Discuss qualitatively how you would expect this force to depend on θ.

(*b*) Compute the force for the angles θ = 0°, 10°, 20°, 30°, 40°, 50°, and 60°, and make a plot of *F* versus θ for *mg* = 400 N. From your plot, at what angle is it most efficient to apply the force to move the block?

(*a*) *F* will decrease with increasing θ for small values of θ since the normal component diminishes; it will reach a minimum and then increase as the tangential component of *F* decreases.

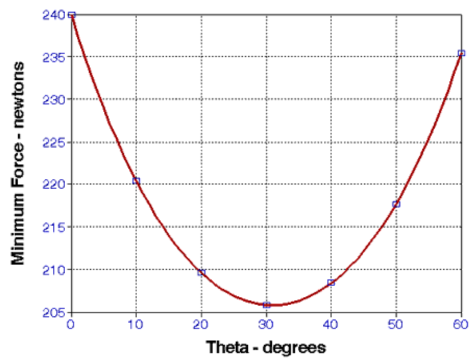
(*b*) The expression for *F* is given in Problem 5-17

$$F = \frac{\mu_s m g}{\cos\theta + \mu_s \sin\theta} \; 0$$

θ (degrees)	0	10	20	30	40	50	60
F (N)	240	220	210	206	208	218	235

A plot of *F* versus θ is shown here.

From the graph it appears that *F* is a minimum at θ = 30°.



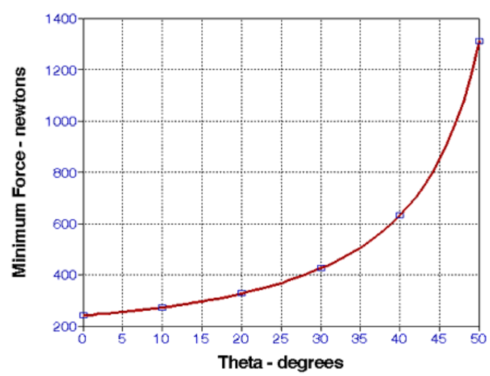
33* ·· Answer the same questions as in Problem 32, only this time with a force ***F*** that pushes down on the block in Figure 5-44 at an angle θ with the horizontal.

(*a*) As in Problem 5-17, replace θ by -θ in the expression for *F*. One expects that *F* will increase with increasing magnitude of the angle since the normal component increases and tangential component decreases.

(*b*)

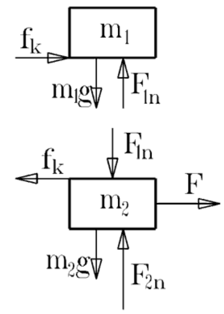
θ (degrees)	0	-10	-20	-30	-40	-50	-60
F (N)	240	272	327	424	631	1310	diverged

A plot of *F* versus the magnitude of θ is shown



34 ·· A 100-kg mass is pulled along a frictionless surface by a horizontal force **F** such that its acceleration is 6 m/s^2 (Figure 5-45). A 20-kg mass slides along the top of the 100-kg mass and has an acceleration of 4 m/s^2. (It thus slides backward relative to the 100-kg mass.) (*a*) What is the frictional force exerted by the 100-kg mass on the 20-kg mass? (*b*) What is the net force acting on the 100-kg mass? What is the force *F*? (*c*) After the 20-kg mass falls off the 100-kg mass, what is the acceleration of the 100-kg mass? (Assume that the force *F* does not change.)

(*a*) 1. Draw the free-body diagram for the masses.



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$. Note that by Newton's thirdlaw, the normal reaction force, F_{n1}, and the friction force acts on both masses but in opposite directions.

$f_k = m_1 a_1$ (1)

$F_{n1} - m_1 g = 0$ (2)

$F - f_k = m_2 a_2 = F_{net}$ (3)

3. Evaluate f_k from (1) — $f_k = (20 \times 4)$ N = 80 N

(*b*) Evaluate F and F_{net} from (3) — $F_{net} = (100 \times 6)$ N = 600 N; $F = 680$ N

(*c*) Use $F = ma$ — $a = (680/100)\ \mathrm{m/s^2} = 6.80\ \mathrm{m/s^2}$

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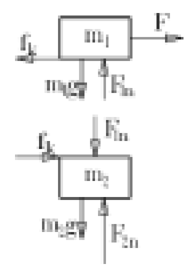
35 ·· A 60-kg block slides along the top of a 100-kg block with an acceleration of 3 m/s^2 when a horizontal force ***F*** of 320 N is applied, as in Figure 5-46. The 100-kg block sits on a horizontal frictionless surface, but there is friction between the two blocks. (*a*) Find the coefficient of kinetic friction between the blocks. (*b*) Find the acceleration of the 100-kg block during the time that the 60-kg block remains in contact.

(*a*) 1. The solution is similar to that of the previous problem except that now the force F acts on the upper mass m_1. The corresponding equations are listed.

$F - f_k = m_1 a_1$ (1)

$F_n - m_1 g = 0$ (2)

$f_k = m_2 a_2$ (3)



2. Replace f_k in (1) by (4) and solve for μ_k

$f_k = \mu_k F_n = \mu_k m_1 g$ (4)

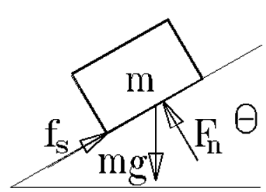
$\mu_k = (F - m_1 a_1)/m_1 g$; $\mu_k = 0.238$

(*b*) From (3) and (4) $a_2 = \mu_k m_1 g/m_2$ — $a_2 = 1.4\ \mathrm{m/s^2}$

---

36 ·· The coefficient of static friction between a rubber tire and the road surface is 0.85. What is the maximum acceleration of a 1000-kg four-wheel-drive truck if the road makes an angle of 12° with the horizontal and the truck is (*a*) climbing, and (*b*) descending?

(*a*) 1. Draw the free-body diagram



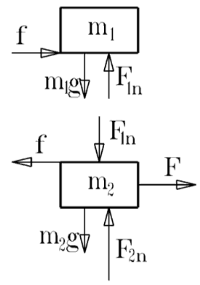
2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $\mu_s mg \cos\theta - mg \sin\theta = ma_{max}$

Solve for and find a_{max} — $a_{max} = g(\mu_s \cos\theta - \sin\theta)$; $a_{max} = 6.12\ \mathrm{m/s^2}$

(*b*) Replace θ by $-\theta$. — $a_{max} = g(\mu_s \cos\theta + \sin\theta)$; $a_{nax} = 10.2\ \mathrm{m/s^2}$

37* ·· A 2-kg block sits on a 4-kg block that is on a frictionless table (Figure 5-47). The coefficients of friction between the blocks are $\mu_s = 0.3$ and $\mu_k = 0.2$. (*a*) What is the maximum force F that can be applied to the 4-kg block if the 2-kg block is not to slide? (*b*) If F is half this value, find the acceleration of each block and the force of friction acting on each block. (*c*) If F is twice the value found in (*a*), find the acceleration of each block.

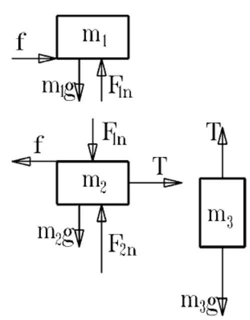
(*a*) 1. Draw the free-body diagram



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$	$f_{s,max} = m_1 a_{max}$; $F_{n1} = m_1 g$; $F_{max} - f_{s,max} = m_2 a_{max}$
3. Use $f_{s,max} = \mu_s F_{n1}$ and solve for a_{max} and F_{max}	$a_{max} = \mu_s g$; $F_{max} = (m_1+m_2)g\mu_s$;
4. Evaluate a_{max} and F_{max}	$a_{max} = 2.94\text{ m/s}^2$, $F_{max} = 17.7$ N
(c) 1. The blocks move as a unit. The force on m_1 is $m_1 a = f_s$.	$a = F/(m_1 + m_2)$; $a = 2.95\text{ m/s}^2$
	$f_s = (2.95 \times 2)\text{ N} = 5.9\text{ N}$
(*c*) 1. If $F = 2F_{max}$ then m_1 slips on m_2.	$f = f_k = \mu_k m_1 g$
2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$	$m_1 a_1 = f_k = \mu_k m_1 g$; $m_2 a_2 = F - \mu_k m_1 g$
3. Solve for and evaluate a_1 and a_2 for $F = 35.4$ N	$a_1 = \mu_k g$; $a_2 = (F - \mu_k g m_1)/m_2$;
	$a_1 = 1.96\text{ m/s}^2$, $a_2 = 7.87\text{ m/s}^2$

38 ·· In Figure 5-48, the mass $m_2 = 10$ kg slides on a frictionless table. The coefficients of static and kinetic friction between m_2 and $m_1 = 5$ kg are $\mu_s = 0.6$ and $\mu_k = 0.4$. (*a*) What is the maximum acceleration of m_1? (*b*) What is the maximum value of m_3 if m_1 moves with m_2 without slipping? (*c*) If $m_3 = 30$ kg, find the acceleration of each body and the tension in the string.

The free-body diagrams for m_1 and m_2 are identical to those of the previous problem. Now the force F arises from the tension T in the string supporting m_3, as shown.



(*a*) See Problem 5-37

(*b*) 1. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$

2. Solve for and evaluate m_3

(d) 1. For $m_3 = 30$ kg, m_1 will slide on m_2. Follow the procedure of Problem 5-37(*c*). Note that $a_3 = a_2$

2. Add the equations involving T to find a_2

3. Evaluate a_1 and T using equation (1)

$a_{\max} = \mu_s g$; $a_{\max} = 5.89\ \mathrm{m/s^2}$

$T = (m_1 + m_2)a_{\max}$; $m_3 g - T = m_3 a_{\max}$

$m_3 = \mu_s(m_1 + m_2)/(1 - \mu_s)$; $m_3 = 22.5$ kg

$m_1 a_1 = f_k = \mu_k m_1 g$; $m_2 a_2 = T - \mu_k m_1 g$; (1)

$m_3 a_3 = m_3 g - T = m_3 a_2$ (2)

$a_2 = (m_3 - m_1\mu_k)g/(m_2 + m_3)$; $a_2 = a_3 = 6.87\ \mathrm{m/s^2}$

$a_1 = (0.4 \times 9.81)\ \mathrm{m/s^2} = 3.92\ \mathrm{m/s^2}$; $T = 88.3$ N

---

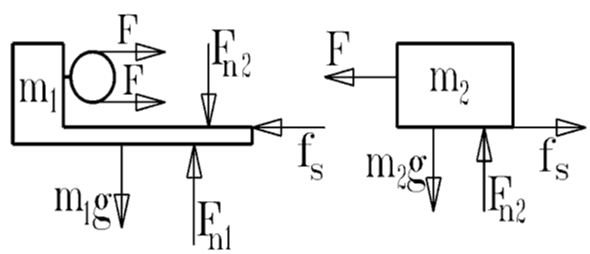
39 ··· A box of mass m rests on a horizontal table. The coefficient of static friction is μ_s. A force $\boldsymbol{F}$ is applied at an angle θ as shown in Problem 5-32. (*a*) Find the force F needed to move the box as a function of the angle θ. (*b*) At the angle θ for which this force is a minimum, the slope $dF/d\theta$ of the curve F versus θ is zero. Compute $dF/d\theta$ and show that this derivative is zero at the angle θ that obeys $\tan\theta = \mu_s$. Compare this general result with that obtained in Problem 5-32.

The expression for F was obtained previously: $F = \mu_s mg/(\cos\theta + \mu_s \sin\theta)$. F is a minimum when the denominator is a maximum. Differentiate $(\cos\theta + \mu_s\sin\theta)$ and set to 0. $(d/d\theta)(\cos\theta + \mu_s\sin\theta) = -\sin\theta + \mu_s\cos\theta = 0$. Solve for θ: $\theta = \tan^{-1}\mu_s$. For $\mu_s = 0.6$, $\theta = 31^\circ$, in agreement with the result of Problem 5-32.

---

40 ··· A 10-kg block rests on a 5-kg bracket like the one shown in Figure 5-49. The 5-kg bracket sits on a frictionless surface. The coefficients of friction between the 10-kg block and the bracket on which it rests are $\mu_s = 0.40$ and $\mu_k = 0.30$. (*a*) What is the maximum force F that can be applied if the 10-kg block is not to slide on the bracket? (*b*) What is the corresponding acceleration of the 5-kg bracket?

(*a*), (*b*) 1. Draw the free-body diagrams for the two objects. The net force acting on m_2 in the direction of motion is $f_s - F$. $a_{2,\max} = f_{s,\max} = \mu_s F_{n2}$ and since m_2 does not move relative to m_1, this is also the acceleration of m_1.



2. Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$

3. Use $f_s = \mu_s F_{n2}$ and solve for $a = a_{\max}$

4. Solve for $F = F_{\max}$

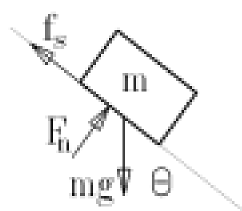
$F_{n2} = m_2 g$; $2F - f_s = m_1 a$; $f_s = m_2 a$

$a_{\max} = \mu_s m_2 g/(m_1 + 2m_2)$; $a_{\max} = 1.57\ \mathrm{m/s^2}$

$F_{\max} = m_2(\mu_s g - a_{\max})$; $F_{\max} = 23.5$ N

41* ··· Lou has set up a kiddie ride at the Winter Ice Fair. He builds a right-angle triangular wedge, which he intends to push along the ice with a child sitting on the hypotenuse. If he pushes too hard, the kid will slide up and over the top, and Lou could be looking at a lawsuit. If he doesn't push hard enough, the kid will slide down the wedge, and the parents will want their money back. If the angle of inclination of the wedge is 40°, what are the minimum and maximum values for the acceleration that Lou must achieve? Use m for the child's mass, and μ_s for the coefficient of static friction between the child and the wedge.

1. Draw the free-body diagam. The diagram is for finding a_{min}; $f_s = f_{s,max} = \mu_s F_n$ and points upward. To find a_{max}, reverse direction of f_s.



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ $\quad F_n\sin\theta - \mu_s F_n\cos\theta = ma;\ F_n\cos\theta + \mu_s F_n\sin\theta - mg = 0$
3. Use the second equation to solve for F_n $\quad F_n = mg/(\cos\theta + \mu_s\sin\theta)$
4. Substitute F_n into first equation and solve for $a = a_{min}$

$$a_{min} = g\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta} 0$$

5. Reverse the direction of f_s and follow the same procedure to find a_{max}.

$$a_{max} = g\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} 0$$

42 ··· A block of mass 0.5 kg rests on the inclined surface of a wedge of mass 2 kg, as in Figure 5-50. The wedge is acted on by a horizontal force **F** and slides on a frictionless surface. (*a*) If the coefficient of static friction between the wedge and the block is $\mu_s = 0.8$, and the angle of the incline is 35°, find the maximum and minimum values of F for which the block does not slip. (*b*) Repeat part (*a*) with $\mu_s = 0.4$.

(*a*) Use results of Problem 5-41 for a_{min} and a_{max}. Then set $F = m_{tot}a$. Substitute numerical values. $\quad a_{min} = -0.627\ \text{m/s}^2$, $F_{min} = -1.57$ N; (accelerate backward)
$a_{max} = 33.5\ \text{m/s}^2$, $F_{max} = 83.7$ N

(*b*) Repeat (*a*) with $\mu_s = 0.4$ $\quad F_{min} = 6.49$ N; $F_{max} = 37.5$ N

43 · True or false: An object cannot move in a circle unless there is a net force acting on it.

True; it requires centripetal force.

44 · An object moves in a circle counterclockwise with constant speed (Figure 5-51). Which figure shows the correct velocity and acceleration vectors?

(*c*)

45* · A particle is traveling in a vertical circle at constant speed. One can conclude that the ________ is constant. (*a*) velocity (*b*) acceleration (*c*) net force (*d*) apparent weight (*e*) none of the above

(*e*)

---

46 · An object travels with constant speed v in a circular path of radius r. (*a*) If v is doubled, how is the acceleration a affected? (*b*) If r is doubled, how is a affected? (*c*) Why is it impossible for an object to travel around a perfectly sharp angular turn?

(*a*) a v^2; a is quadrupled. (*b*) a $1/r$; a is halved. (*c*) Would require an infinite centripetal force (r = 0).

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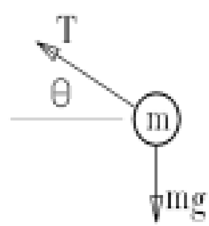
47 · A boy whirls a ball on a string in a horizontal circle of radius 0.8 m. How many revolutions per minute must the ball make if the magnitude of its centripetal acceleration is to be the same as the free-fall acceleration due to gravity g?

$a_c = \omega^2 r = g$; $\omega = \sqrt{g/r} = \sqrt{9.81/0.8}$ rad/s = 3.5 rad/s = 33.4 rpm

---

48 · A 0.20-kg stone attached to a 0.8-m long string is rotated in a horizontal plane. The string makes an angle of 20° with the horizontal. Determine the speed of the stone.

1. Draw the free-body diagram. Note that $a_y = 0$ and $a_x = v^2/r$. The radius of the circle is $r = L\cos\theta$, where L is the length of the string.



2. Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$ — $T\sin\theta = mg$; $T\cos\theta = mv^2/(L\cos\theta)$

3. Solve for and evaluate v. — $v = \sqrt{L\,g\cos\theta\ \cot\theta}$ 0; v = 4.5 m/s

---

49* · A 0.75-kg stone attached to a string is whirled in a horizontal circle of radius 35 cm as in the conical pendulum of Example 5-10. The string makes an angle of 30° with the vertical. (*a*) Find the speed of the stone. (*b*) Find the tension in the string.

This problem is identical to Problem 5-48; since the angle θ is with respect to the vertical, the expressions for v and T must be changed accordingly.

(*a*), (*b*) Write v and T in terms of θ and r — $v = \sqrt{r\,g\ \tan\theta}$ 0; $T = mg/\cos\theta$

Evaluate v and T — v = 1.41 m/s; T = 8.5 N

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50 ·· A stone with a mass m = 95 g is being whirled in a horizontal circle at the end of a string that is 85 cm long. The length of required time for the stone to make one complete revolution is 1.22 s. The angle that the string makes with the horizontal is _____. (*a*) 52° (*b*) 46° (*c*) 26° (*d*) 23° (*e*) 3°

$v^2/Lg = \cos\theta\cot\theta$ (see Problem 5-48); substitute $v = r\omega = \omega L\cos\theta$ and obtain $\sin\theta = g/L\omega^2$.

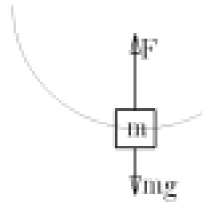
$\omega = (2\pi/1.22)$ rad/s.

$\theta = \sin^{-1}[(9.81 \times 1.22^2)/(0.85 \times 4\pi^2)] = 25.8°$; (*c*) is correct.

---

51 ·· A pilot of mass 50 kg comes out of a vertical dive in a circular arc such that her upward acceleration is 8.5g. (*a*) What is the magnitude of the force exerted by the airplane seat on the pilot at the bottom of the arc? (*b*) If the speed of the plane is 345 km/h, what is the radius of the circular arc?

(*a*) 1. Draw the free-body diagram



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $F - mg = ma$

3. Solve for and evaluate F — $F = 9.5mg = 4660$ N

(*b*) $r = v^2/a_c$; evaluate for $a_c = 8.5g$, $v = 95.8$ m/s — $r = 110$ m

---

52 ·· A 65-kg airplane pilot pulls out of a dive by following the arc of a circle whose radius is 300 m. At the bottom of the circle, where her speed is 180 km/h, (*a*) what are the direction and magnitude of her acceleration? (*b*) What is the net force acting on her at the bottom of the circle? (*c*) What is the force exerted on the pilot by the airplane seat?

(*a*) 1. See Problem 5-51 for the free-body diagram.

2. $a = a_c = v^2/r$ — $a = (50^2/300)$ m/s^2 = 8.33 m/s^2, directed up

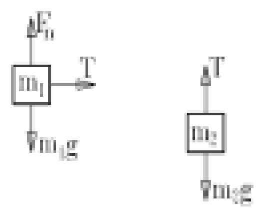
(*b*) $F_{net} = ma$ — $F_{net} = (65 \times 8.33)$ N = 542 N, directed up

(*c*) $F = mg + F_{net}$ — $F = (542 + 65 \times 9.81)$ N = 1179 N, directed up

---

53* ·· Mass m_1 moves with speed v in a circular path of radius R on a frictionless horizontal table (Figure 5-52). It is attached to a string that passes through a frictionless hole in the center of the table. A second mass m_2 is attached to the other end of the string. Derive an expression for R in terms of m_1, m_2, and v.

1. Draw the free-body diagrams for the two masses



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $T = m_1v^2/R$; $T - m_2g = 0$

3. Solve for R — $R = (m_1/m_2)v^2/g$

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54 ·· In Figure 5-53, particles are shown traveling counterclockwise in circles of radius 5 m. The acceleration vectors are indicated at three specific times. Find the values of v and dv/dt for each of these times.

(*a*) The acceleration is radial; $a = a_c = v^2/r$; $v = (a_c r)^{1/2} = (20 \times 5)^{1/2}$ m/s = 10 m/s. $dv/dt = 0$

(*b*) $\boldsymbol{a}$ has radial and tangential components. $a_r = a_c = (30 \cos 30^\circ)$ m/s^2; $v = (a_c r)^{1/2}$; for $r = 5$ m, $v = 11.4$ m/s. The tangential acceleration is $a \sin 30^\circ = 15$ m/s^2 = dv/dt.

(c) Here a_c = (50 cos 45°) m/s^2 = v^2/r. For r = 5 m, v = 13.3 m/s. The tangential acceleration is directed opposite to $\boldsymbol{v}$ and its magnitude is (50 sin 45°) m/s^2 = 35.4 m/s^2. Hence, dv/dt = -35.4 m/s^2.

55 ·· A block of mass m_1 is attached to a cord of length L_1, which is fixed at one end. The block moves in a horizontal circle on a frictionless table. A second block of mass m_2 is attached to the first by a cord of length L_2 and also moves in a circle, as shown in Figure 5-54. If the period of the motion is T, find the tension in each cord.

1. Draw the free-body diagrams for the two blocks
   Note that there is no vertical motion.

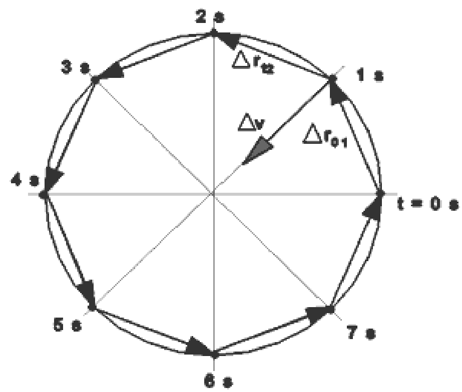
2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ to each mass — $T_2 = m_2(L_1 + L_2)\omega^2 = m_2(L_1 + L_2)(2\pi/T)^2$

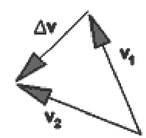
   $T_1 - T_2 = m_1L_1(2\pi/T)^2$

3. Solve for T_1 — $T_1 = [m_1L_1 + m_2(L_1 + L_2)](2\pi/T)^2$

56 ·· A particle moves with constant speed in a circle of radius 4 cm. It takes 8 s to make a complete trip. Draw the path of the particle to scale, and indicate the particle's position at 1-s intervals. Draw displacement vectors for each interval. These vectors also indicate the directions for the average-velocity vectors for each interval. Find graphically the change in the average velocity $\Delta\boldsymbol{v}$ for two consecutive 1-s intervals. Compare $\Delta\boldsymbol{v}/\Delta t$, measured in this way, with the instantaneous acceleration computed from $a = v^2/r$.

The path of the particle and its position at 1 s intervals are shown. The displacement vectors are also shown. The velocity vectors for the average velocities in the first and second intervals are along Δ in the lower diagra $\Delta v = 2v_1 \sin 22.5^\circ$. Also $\Delta r = 2r\sin 22.5^\circ = 3.06$ cm. T and $\Delta v/\Delta t = 2.34$ cm/s^2. The instantaneous speed is $2\pi r/T = \pi$ cm/s and the instantaneous acceleration is then $v^2/r = (3.14^2/4)$ cm/s^2 = 2.47 cm/s^2.





57* ·· A man swings his child in a circle of radius 0.75 m, as shown in the photo. If the mass of the child is 25 kg and the child makes one revolution in 1.5 s, what are the magnitude and direction of the force that must be exerted by the man on the child? (Assume the child to be a point particle.)

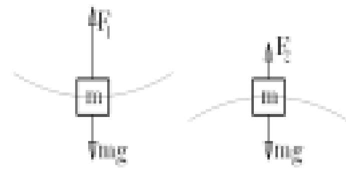
1. See Problem 5-49. In this problem T stands for the period.
2. $\tan\theta = v^2/rg = r\omega^2/g = 4\pi^2 r/gT^2$ (see Problem 5-49) — $\tan\theta = (4\pi^2 \times 0.75)/(9.81 \times 1.5^2) = 1.34$; $\theta = 53.3^\circ$
3. $F = mg/\cos\theta$ — $F = (25 \times 9.81/\cos 53.3^\circ) = 410$ N

58 ·· The string of a conical pendulum is 50 cm long and the mass of the bob is 0.25 kg. Find the angle between the string and the horizontal when the tension in the string is six times the weight of the bob. Under those conditions, what is the period of the pendulum?

1. See Problem 5-48 for the free-body diagram and the relevant equations.
2. $\sin\theta = mg/T$; solve for and evaluate θ — $\theta = \sin^{-1}(1/6) = 9.6^\circ$
3. $T = 2\pi r/v = 2\pi r/\sqrt{L\,g\cos\theta\ \cot\theta}$; evaluate T — $T = 2\pi\sqrt{L\sin\theta/g}\ 0 = 0.58$ s

59 ·· Frustrated with his inability to make a living through honest channels, Lou sets up a deceptive weight-loss scam. The trick is to make insecure customers believe they can "think those extra pounds away" if they will only take a ride in a van that Lou claims to be "specially equipped to enhance mental-mass fluidity." The customer sits on a platform scale in the back of the van, and Lou drives off at constant speed of 14 m/s. Lou then asks the customer to "think heavy" as he drives through the bottom of a dip in the road having a radius of curvature of 80 m. Sure enough, the scale's reading increases, until Lou says, "Now think light," and drives over the crest of a hill having a radius of curvature of 100 m. If the scale reads 800 N when the van is on level ground, what is the range of readings for the trip described here?

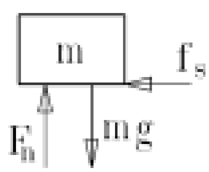
1. Draw the free-body diagrams for each case. Passing through the dip, $\boldsymbol{a}_c$ is upward; driving over the crest, $\boldsymbol{a}_c$ is downward. The apparent weights are F_1 and F_2, respectively.



2. Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$ — $F_1 - mg = mv^2/r_1$ $\quad F_2 - mg = -mv^2/r_2$
3. Evaluate F_1 and F_2. — $F_1 = 1000$ N; $F_2 = 640$ N

60 ·· A 100-g disk sits on a horizontally rotating turntable. The turntable makes one revolution each second. The disk is located 10 cm from the axis of rotation of the turntable. (*a*) What is the frictional force acting on the disk? (*b*) The disk will slide off the turntable if it is located at a radius larger than 16 cm from the axis of rotation. What is the coefficient of static friction?

(*a*) 1. Draw the free-body diagram.



2. Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$ — $F_n = mg$; $f_s = mr\omega^2 = mr(2\pi/T)^2$
3. Evaluate f_s — $f_s = 0.395$ N

(*b*) For $r = 0.16$ m, $f_s = \mu_s F_n$. Find μ_s — $\mu_s = 4\pi^2 r/gT^2 = 0.644$

61* ·· A tether ball of mass 0.25 kg is attached to a vertical pole by a cord 1.2 m long. Assume the cord attaches to the center of the ball. If the cord makes an angle of 20° with the vertical, then (*a*) what is the tension in the cord? (*b*) What is the speed of the ball?

This problem is identical to Problem 5-48, except that the angle θ is now with respect to the vertical.

Consequently, the relevant equations are: $T\cos\theta = mg$ and $v = \sqrt{L g \sin\theta \tan\theta}$ 0. Substituting the appropriate numerical values one obtains (*a*) $T = 2.61$ N, (*b*) $v = 1.21$ m/s

62 ·· An object on the equator has an acceleration toward the center of the earth because of the earth's rotation and an acceleration toward the sun because of the earth's motion along its orbit. Calculate the magnitudes of both accelerations, and express them as fractions of the free-fall acceleration due to gravity g.

1. Evaluate ω_R (rotation) and ω_O (orbital motion) — $\omega_R = 2\pi/(24 \times 60 \times 60)$ rad/s $= 7.27 \times 10^{-5}$ rad/s; $\omega_O = \omega_R/365 = 19.9 \times 10^{-8}$ rad/s
2. Evaluate $R_e\omega_R^2$ and $R_o\omega_O^2$ (see Appendix B on page AP-3) — $a_R = 3.37 \times 10^{-2}$ m/s^2 $= 3.44 \times 10^{-3}g$; $a_O = 5.95 \times 10^{-3}$ m/s^2 $= 6.1 \times 10^{-4}g$

63 ·· A small bead with a mass of 100 g slides along a semicircular wire with a radius of 10 cm that rotates about a vertical axis at a rate of 2 revolutions per second, as shown in Figure 5-55. Find the values of θ for which the bead will remain stationary relative to the rotating wire.

The semicircular wire of radius 10 cm limits the motion of the bead in the same manner as would a 10-cm string attached to the bead and fixed at the center of the semicircle. Consequently, we can use the expression for T derived in Problem 5-58, $T = 2\pi\sqrt{L\sin\theta/g}$, where here θ is the angle with respect to the horizontal. Thus, we shall use $T = 2\pi\sqrt{L\cos\theta/g}$. Solving for θ we obtain $\theta = \cos^{-1}(T^2g/4\pi^2L) = 51.6°$.

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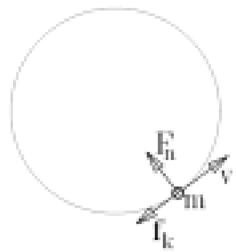
64 ··· Consider a bead of mass m that is free to move on a thin, circular wire of radius r. The bead is given an initial speed v_0, and there is a coefficient of kinetic friction μ_k. The experiment is performed in a spacecraft drifting in space. Find the speed of the bead at any subsequent time t.

1. Draw the free-body diagram. Note that the acceleration of the bead has two components, the radial component perpendicular to ***v***, and a tangential component due to friction directed opposite to ***v***.



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $F_n = mv^2/r$; $f_k = \mu_k mv^2/r = -m(dv/dt)$
3. Rewrite the differential equation — $dv/v^2 = -(\mu_k/r)dt$
4. Integrate the differential equation; the limits on v and t are v_0 and v, and 0 and t, respectively.

$$-\left(\frac{1}{v}-\frac{1}{v_0}\right) = -\left(\frac{\mu_k}{r}\right)t\ ;\ v = v_0\left(\frac{1}{1+(\mu_k v_0/r)t}\right)$$

65* ··· Revisiting the previous problem, (*a*) find the centripetal acceleration of the bead. (*b*) Find the tangential acceleration of the bead. (*c*) What is the magnitude of the resultant acceleration?

(*a*) Use the result of Problem 5-64 — $a_c = v^2/r = \dfrac{v_0^2}{r}\left(\dfrac{1}{1+(\mu_k v_0/r)t}\right)^2$

(*b*) $a_t = -\mu_k v^2/r$ — $a_t = -\mu_k a_c$

(*c*) $a = (a_c^2 + a_t^2)^{1/2}$ — $a = a_c(1+\mu_k^2)^{1/2}$, where a_c is given above.

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66 · A block is sliding on a frictionless surface along a loop-the-loop, as shown in Figure 5-56. The block is moving fast enough that it never loses contact with the track. Match the points along the track to the appropriate free-body diagrams (Figure 5-57).

A : 3; B : 4; C : 5; D : 2.

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67 · A person rides a loop-the-loop at an amusement park. The cart circles the track at constant speed. At the top of the loop, the normal force exerted by the seat equals the person's weight, mg. At the bottom of the loop, the force exerted by the seat will be _____. (*a*) 0 (*b*) mg (*c*) $2mg$ (*d*) $3mg$ (*e*) greater than mg, but the exact value cannot be calculated from the information given

At the top, $mv^2/r = 2mg$. At the bottom, $F_n = mg + 2mg = 3mg$. (*d*) is correct.

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68 · The radius of curvature of a loop-the-loop roller coaster is 12.0 m. At the top of the loop, the force that the seat exerts on a passenger of mass *m* is 0.4*mg*. Find the speed of the roller coaster at the top of the loop.

$mv^2/r = 1.4mg$; $v = (1.4rg)^{1/2} = 12.8$ m/s

69* · Realizing that he has left the gas stove on, Aaron races for his car to drive home. He lives at the other end of a long, unbanked curve in the highway, and he knows that when he is traveling alone in his car at 40 km/h, he can just make it around the curve without skidding. He yells at his friends, "Get in the car! With greater mass, I can take the curve at higher speed!" Carl says, "No, that will make you skid at even lower speed." Bonita says, "The mass does not matter. Just get going!" Who is right?

Bonita is right.

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70 · A car speeds along the curved exit ramp of a freeway. The radius of the curve is 80 m. A 70-kg passenger holds the arm rest of a car door with a 220-N force to keep from sliding across the front seat of the car. (Assume the exit ramp is not banked and ignore friction with the car seat.) What is the car's speed? (*a*) 16 m/s (*b*) 57 m/s (*c*) 18 m/s (*d*) 50 m/s (*e*) 28 m/s

$mv^2/r = F$; $v = (Fr/m)^{1/2}$; $v = 15.9$ m/s. (*a*) is correct.

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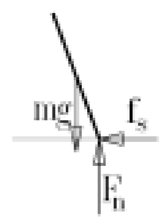
71 ··· Suppose you ride a bicycle on a horizontal surface in a circle with a radius of 20 m. The resultant force exerted by the road on the bicycle (normal force plus frictional force) makes an angle of 15° with the vertical. (*a*) What is your speed? (*b*) If the frictional force is half its maximum value, what is the coefficient of static friction?

(*a*) 1. Draw the free-body diagram



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $F_n = mg$; $f_s = mv^2/r$

3. $\boldsymbol{F}_n + \boldsymbol{f}_s$ makes an angle of 15° with vertical — $v^2/rg = \tan 15°$

4. Solve for *v* — $v = (20 \times 9.81 \times \tan 15°)^{1/2} = 7.25$ m/s

(*b*) $\mu_s = f_{s,max}/F_n = 2f_s/F_n$ — $\mu_s = 2v^2/rg = 2\tan 15° = 0.536$

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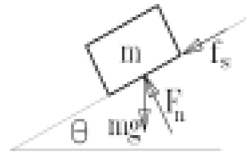
72 · A 750-kg car travels at 90 km/h around a curve with a radius of 160 m. What should the banking angle of the curve be so that the only force between the pavement and tires of the car is the normal reaction force?

1. See Example 5-12. — $\theta = \tan^{-1}(v^2/rg) = 21.7°$.

73* ·· A curve of radius 150 m is banked at an angle of 10°. An 800-kg car negotiates the curve at 85 km/h without skidding. Find (*a*) the normal force on the tires exerted by the pavement, (*b*) the frictional force exerted by the pavement on the tires of the car, and (*c*) the minimum coefficient of static friction between the pavement and tires.

(*a*), (*b*) 1. Draw the free-body diagram

2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$



3. Multiply (1) by $\sin\theta$, (2) by $\cos\theta$ and add
4. Evaluate F_n and use (2) to evaluate f_s

(*c*) $\mu_{s,\min} = f_s/F_n$

$F_n\sin\theta + f_s\cos\theta = mv^2/r$ (1)

$F_n\cos\theta - f_s\sin\theta = mg$ (2)

$F_n = (mv^2/r)\sin\theta + mg\cos\theta$

$F_n = 8245$ N; $f_s = 1565$ N

$\mu_{s,\min} = 0.19$

74 ·· On another occasion, the car in the previous problem negotiates the curve at 38 km/h. Find (*a*) the normal force exerted on the tires by the pavement, and (*b*) the frictional force exerted on the tires by the pavement.

Proceed as in the previous problem. One obtains (*a*) $F_n = 7832$ N, and (*b*) $f_s = -766$ N (f_s points up along the plane.)

75 ··· A civil engineer is asked to design a curved section of roadway that meets the following conditions: With ice on the road, when the coefficient of static friction between the road and rubber is 0.08, a car at rest must not slide into the ditch and a car traveling less than 60 km/h must not skid to the outside of the curve. What is the minimum radius of curvature of the curve and at what angle should the road be banked?

1. The free-body diagram for the car at rest is that of Problem 5-73; for the car at 60 km/h, reverse f_s. In each case we require that $f_s = f_{s,\max} = \mu_s F_n$.
2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ for $v = 0$
3. Solve (2) for θ and evaluate
4. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ for $v \neq 0$
5. Substitute numerical values into (1*a*) and (2*a*)
6. Evaluate r for $v = 16.67$ m/s

$F_n(\cos\theta + \mu_s\sin\theta) = \text{mg}$ (1); $F_n(\mu_s\cos\theta - \sin\theta) = 0$ (2)

$\theta = \tan^{-1}(\mu_s)$; $\theta = \tan^{-1}(0.08) = 4.57°$

$F_n(\cos\theta - \mu_s\sin\theta) = mg$ (1*a*);

$F_n(\mu_s\cos\theta + \sin\theta) = mv^2/r$ (2*a*)

$0.9904F_n = mg$; $0.1595F_n = mv^2/r$

$r = 176$ m

76 ··· A curve of radius 30 m is banked so that a 950-kg car traveling 40 km/h can round it even if the road is so icy that the coefficient of static friction is approximately zero. Find the range of speeds at which a car can travel around this curve without skidding if the coefficient of static friction between the road and the tires is 0.3.

This problem is similar to the preceding problem, and we shall use the free-body diagram of Problem 5-73.

1. Determine the banking angle
2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ for $v = v_{\min}$ (diagram $v = 0$ of Problem 5-75)

$\theta = \tan^{-1}(v^2/rg) = 22.8°$

$F_n(\cos\theta + \mu_s\sin\theta) = \text{mg}$; $F_n(\mu_s\cos\theta - \sin\theta) = mv_{\min}^2/r$

3. Evaluate for $\theta = 22.8^\circ$, $\mu_s = 0.3$. $1.038F_n = mg$; $0.1102F_n = mv_{min}^2/r$; $v_{min}^2 = 0.106rg$

4. Evaluate v_{mn} $v_{min} = 5.59$ m/s = 20.1 km/h

5. Repeat steps 3, 4 using (1*a*) and (2*a*) of Problem 5-75 $v_{max} = 15.57$ m/s = 56.1 km/h

77* · How would you expect the value of *b* for air resistance to depend on the density of air?

The constant *b* should increase with density as more air molecules collide with the object as it falls.

78 · True or false: The terminal speed of an object depends on its shape.

True.

79 · As a skydiver falls through the air, her terminal speed (*a*) depends on her mass. (*b*) depends on her orientation as she falls. (*c*) equals her weight. (d) depends on the density of the air. (*e*) depends on all of the above.

(*a*), (*b*), and (*d*)

80 · What are the dimensions and SI units of the constant *b* in the retarding force bv^n if (*a*) $n = 1$, and (*b*) $n = 2$?

(*a*) For $n = 1$, [b] = [F]/[v] = $[ML/T^2]/[L/T]$ = [M/T], kg/s; (*b*) for $n = 2$, [b] = $[ML/T^2]/[L^2/T^2]$ = [M/L], kg/m

81* · A small pollution particle settles toward the earth in still air with a terminal speed of 0.3 mm/s. The particle has a mass of 10^{-10} g and a retarding force of the form *bv*. What is the value of *b*?

When $v = v_t$, $bv = mg$, $b = mg/v_t$ $b = (10^{-13} \times 9.81/3 \times 10^{-4})$ kg/s = 3.27×10^{-9} kg/s

82 · A Ping-Pong ball has a mass of 2.3 g and a terminal speed of 9 m/s. The retarding force is of the form bv^2. What is the value of *b*?

For $v = v_t$, $bv_t^2 = mg$, $b = mg/v_t^2$ $b = [(2.3 \times 10^{-3} \times 9.81)/(9^2)]$ kg/m = 2.79×10^{-4} kg/m

83 · A sky diver of mass 60 kg can slow herself to a constant speed of 90 km/h by adjusting her form. (*a*) What is the magnitude of the upward drag force on the sky diver? (*b*) If the drag force is equal to bv^2, what is the value of *b*?

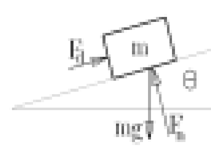
(*a*) Since $a = 0$, $F_d = mg = 589$ N. (*b*) (See Problem 5-82) $b = mg/v_t^2 = (589/25^2)$ kg/m = 0.942 kg/m

84 · Newton showed that the air resistance on a falling object with circular cross section should be approximately $\frac{1}{2}\rho\pi r^2v^2$, where $\rho = 1.2$ kg/m^3, the density of air. Find the terminal speed of a 56-kg sky diver, assuming that his cross-sectional area is equivalent to a disk of radius 0.30 m.

For $v = v_t$, $F_d = mg = \frac{1}{2}\pi\rho r^2v^2$; $v_t = \sqrt{(2mg)/(\rho\pi r^2)}$ 0 = 56.9 m/s

85* ·· An 800-kg car rolls down a very long 6° grade. The drag force for motion of the car has the form $F_d = 100\text{ N} + (1.2\text{ N·s}^2/\text{m}^2)v^2$. What is the terminal velocity of the car rolling down this grade?

1. Draw the free-body diagram. Note that the car moves at constant velocity, i.e., $a = 0$.



2. Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$ $F_d = mg\sin\theta$

3. Use F_d as given, m = 800 kg, and $\theta = 6^\circ$	$(100 + 1.2v^2)$ N = 820 N
4. Evaluate $v = v_t$	v_t = 24.5 m/s = 88.2 km/h

86 ·· While claims of hailstones the size of golf balls may be a slight exaggeration, hailstones are often substantially larger than raindrops. Estimate the terminal velocity of a raindrop and a large hailstone. (See Problem 5-84.)

1. Estimate the radius of a raindrop and a hailstone	Raindrop, r_r = 0.5 mm; hailstone, r_h = 1 cm
2. Evaluate b_r and b_h using $b = 1/2\pi\rho r^2$	$b_r = 4.7 \times 10^{-7}$ kg/m; $b_h = 1.9 \times 10^{-4}$ kg/m
3. Find m_r and m_h using $m = 4\pi r^3\rho/3$	$\rho_r = 10^3$ kg/m^3, $m_r = 5.2 \times 10^{-7}$ kg; $\rho_h = 920$ kg/m^3, $m_h = 3.8 \times 10^{-3}$ kg
4. Find $v_{t,r}$ and $v_{t,h}$ using $v_t = (mg/b)^{1/2}$	$v_{t,r}$ 3 m/s; $v_{t,h}$ 14 m/s

87 ·· (*a*) A parachute creates enough air resistance to keep the downward speed of an 80-kg sky diver to a constant 6 m/s. Assuming the force of air resistance is given by $f = bv^2$, calculate b for this case. (*b*) A sky diver free-falls until his speed is 60 m/s before opening his parachute. If the parachute opens instantaneously, calculate the initial upward force exerted by the chute on the sky diver moving at 60 m/s. Explain why it is important that the parachute takes a few seconds to open.

(*a*) For $v = v_t$, $f = mg = bv_t^2$; solve for and find b	$b = mg/v_t^2 = (80 \times 9.81/36)$ kg/m = 21.8 kg/m
(*b*) Find f	f = 78.48 kN, corresponds to $a = 100g$
This initial acceleration would cause internal damage	

88 ··· An object falls under the influence of gravity and a drag force $F_d = -bv$. (*a*) By applying Newton's second law, show that the acceleration of the object can be written $a = dv/dt = g - (b/m)v$. (*b*) Rearrange this equation to obtain $dv/(v - v_t) = -(g/v_t)dt$, where $v_t = mg/b$. (*c*) Integrate this equation to obtain the exact solution $v = \frac{mg}{b}(1 - e^{-bt/m}) = v_t(1 - e^{-gt/v_t})$ 0. (*d*) Plot v versus t for v_t = 60 m/s.

(*a*) From Newton's second law, $ma = F_{net} = mg - bv$. Divide both sides by m and replace a by dv/dt to obtain the result.

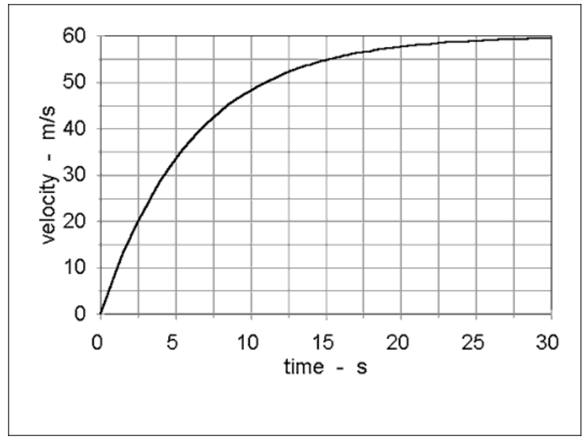
(*b*) Multiply both sides by $-dt/[g - (b/m)v]$; then multiply by (b/m) and replace gm/b by v_t.

(*c*) Integrate over v between the limits of 0 and v; integrate over t between the limits of 0 and t.

$$\int_0^v \frac{dv}{v - v_t} = \ln\left(\frac{v_t - v}{v_t}\right) = \ln(1 - v/v_t) = -\frac{g}{v_t}\int_0^t dt = -\frac{gt}{v_t}$$; take antilogs of both sides to obtain

$1 - v/v_t = e^{-gt/v_t}$. Solving for v gives $v = v_t\left(1 - e^{-gt/v_t}\right)$

(*d*) The graph of v versus t for $v_t = 60$ m/s is shown.



89* ··· Small spherical particles experience a viscous drag force given by Stokes' law: $F_d = 6\pi\eta rv$, where r is the radius of the particle, v is its speed, and η is the viscosity of the fluid medium. (*a*) Estimate the terminal speed of a spherical pollution particle of radius 10^{-5} m and density 2000 kg/m^3. (*b*) Assuming that the air is still and η is 1.8×10^{-5} N·s/m^2, estimate the time it takes for such a particle to fall from a height of 100 m.

Assume a spherical particle. Also, neglect the time required to attain terminal velocity; we will later confirm that this assumption is justified.

(*a*) Using Stokes's law and $m = (4/3)\pi r^3\rho$ solve for v_t — $v_t = (2r^2\rho g)/(9\eta) = 2.42$ cm/s

(*b*) Find the time to fall 100 m at 2.42 cm/s — $t = (10^4 \text{ cm})/(2.42 \text{ cm/s}) = 4.13 \times 10^3 \text{ s} = 1.15$ h

Find time, t', to reach v_t (see Problem 5-88) — $t' \quad 5v_t/g = 12$ ms <<< 1.15 h; neglect of t' is justified

90 ··· An air sample containing pollution particles of the size and density given in Problem 5-89 is captured in a test tube 8.0 cm long. The test tube is then placed in a centrifuge with the midpoint of the test tube 12 cm from the center of the centrifuge. The centrifuge spins at 800 revolutions per minute. Estimate the time required for nearly all of the pollution particles to sediment at the end of the test tube and compare this to the time required for a pollution particle to fall 8 cm under the action of gravity and subject to the viscous drag of air.

1. The effective acceleration is $a_c = r\omega^2$ — $r\omega^2 = [0.12 \times (2\pi \times 800/60)^2]$ m/s^2 = 840 m/s^2 >> g
2. Find $v_t = (2r^2\rho a_c)/(9\eta)$ (See Problem 5-89) — $v_t = (2.42 \text{ cm/s})(840/9.81) \quad 200$ cm/s
3. Find time to move 8 cm — $t = (8/200)$ s = 40 ms
4. Find time to fall under g, i.e., at 2.42 cm/s — $t_g = (8/2.42) \text{ s} \quad 3$ s

91 · The mass of the moon is about 1% that of the earth. The centripetal force that keeps the moon in its orbit around the earth (*a*) is much smaller than the gravitational force exerted by the moon on the earth. (*b*) depends on the phase of the moon. (*c*) is much greater than the gravitational force exerted by the moon on the earth. (*d*) is the same as the gravitational force exerted by the moon on the earth. (*e*) I cannot answer; we haven't studied Newton's law of gravity yet.

(*d*) by Newton's third law.

92 · True or false: Centripetal force is one of the four fundamental forces.

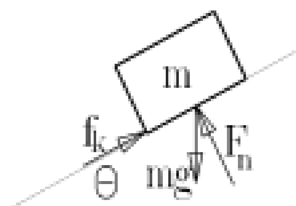
False

93* · On an icy winter day, the coefficient of friction between the tires of a car and a roadway might be reduced to one-half its value on a dry day. As a result, the maximum speed at which a curve of radius R can be safely negotiated is (*a*) the same as on a dry day. (*b*) reduced to 70% of its value on a dry day. (*c*) reduced to 50% of its value on a dry day. (*d*) reduced to 37% of its value on a dry day. (*e*) reduced by an unknown amount depending on the car's mass.

(*b*) $v_{max} = (\mu_s gR)^{1/2}$; therefore $v'_{max} = v_{max}/\sqrt{2}$.

94 · A 4.5-kg block slides down an inclined plane that makes an angle of 28° with the horizontal. Starting from rest, the block slides a distance of 2.4 m in 5.2 s. Find the coefficient of kinetic friction between the block and plane.

1. Draw the free-body diagram



2. $s = {}_{1/2}at^2$; solve for and find a	$a = 2s/t^2 = 0.1775\ \text{m/s}^2$
3. Apply $\Sigma \mathbf{F} = m\mathbf{a}$	$mg \sin\theta - f_k = ma$; $F_n = mg\cos\theta$
4. Set $f_k = \mu_k F_n$ and solve for μ_k	$\mu_k = (g \sin\theta - a)/(g\cos\theta)$
5. Find μ_k for $a = 0.1775\ \text{m/s}^2$, $\theta = 28°$	$\mu_k = 0.51$

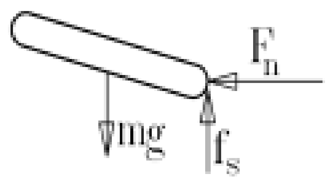
95 · A model airplane of mass 0.4 kg is attached to horizontal string and flies in a horizontal circle of radius 5.7 m. (The weight of the plane is balanced by the upward "lift" force of the air on the wings of the plane.) The plane makes 1.2 revolutions over 4 s. (*a*) Find the speed v of the plane. (*b*) Find the tension in the string.

(*a*) $v = r\omega = 2\pi r/T$	$v = [2\pi \times 5.7/(4/1.2)]\ \text{m/s} = 10.7\ \text{m/s}$
(*b*) F (tension) $= ma = ma_c = mv^2/r$	$F = (0.4 \times 10.7^2/5.7)\ \text{N} = 8.0\ \text{N}$

96 ·· Show with a force diagram how a motorcycle can travel in a circle on the inside vertical wall of a hollow cylinder. Assume reasonable parameters (coefficient of friction, radius of the circle, mass of the motorcycle, or whatever is required), and calculate the minimum speed needed.

We shall take the following values for the numerical calculation: $R = 6.0$ m, $\mu_s = 0.8$.

1. The appropriate free-body diagram is shown. The normal reaction force F_n provides the centripetal force, and the force of static friction, $\mu_s F_n$ keeps the cycle from sliding down the wall.



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$

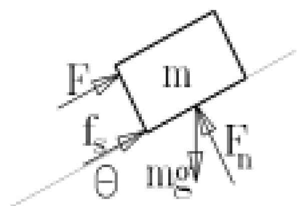
$\mu_s F_n = mg$; $F_n = mv^2/R$

3. Solve for and evaluate $v = v_{min}$

$v_{min} = \sqrt{Rg/\mu_s} = 8.6 \text{ m/s} = 31 \text{ km/h}$

97* ·· An 800-N box rests on a plane inclined at 30° to the horizontal. A physics student finds that she can prevent the box from sliding if she pushes with a force of at least 200 N parallel to the surface. (*a*) What is the coefficient of static friction between the box and the surface? (*b*) What is the greatest force that can be applied to the box parallel to the incline before the box slides up the incline?

(*a*) 1. Draw the free-body diagram.



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $F + f_s - mg \sin\theta = 0$; $F_n = mg\cos\theta$

3. Use $f_{s,max} = \mu_s F_n$ and solve for and find μ_s — $\mu_s = \tan\theta - F/(mg\cos\theta)$; $\mu_s = 0.289$

(*b*) 1. Find $f_{s,max}$ from part (*a*) — $f_{s,max} = mg\sin\theta - F = 400 \text{ N} - 200 \text{ N} = 200 \text{ N}$

2. Reverse the direction of $f_{s,max}$ and evaluate F — $F = mg\sin\theta + f_{s,max} = 400 \text{ N} + 200 \text{ N} = 600 \text{ N}$

98 · The position of a particle is given by the vector $\boldsymbol{r} = -10 \text{ m} \cos\omega t\, \boldsymbol{i} + 10 \text{ m} \sin\omega t\, \boldsymbol{j}$, where $\omega = 2 \text{ s}^{-1}$. (*a*) Show that the path of the particle is a circle. (*b*) What is the radius of the circle? (*c*) Does the particle move clockwise or counterclockwise around the circle? (*d*) What is the speed of the particle? (*e*) What is the time for one complete revolution?

(*a*), (*b*) We need to show that r is constant. $r = \sqrt{r_x^2 + r_y^2} = \sqrt{100(\cos^2\omega t + \sin^2\omega t)} \text{ m} = 10 \text{ m}$.

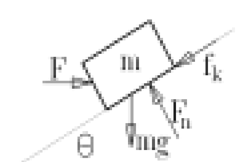
(*c*) Note that at $t = 0$, $x = -10$ m, $y = 0$; at $t = \Delta t$ (Δt small), the particle is at x -10 m, $y = \Delta y$, where Δy is positive. It follows that the motion is clockwise.

(*d*) $\boldsymbol{v} = d\boldsymbol{r}/dt = (10\omega \sin\omega t)\, \boldsymbol{i} \text{ m} + (10\omega\cos\omega t)\, \boldsymbol{j} \text{ m}$; $v = \sqrt{v_x^2 + v_y^2} = 10\omega = 20 \text{ m/s}$

(*e*) $T = 2\pi/\omega = \pi$ s

99 ·· A crate of books is to be put on a truck with the help of some planks sloping up at 30°. The mass of the crate is 100 kg, and the coefficient of sliding friction between it and the plank is 0.5. You and your friends push *horizontally* with a force $\boldsymbol{F}$. Once the crate has started to move, how large must F be in order to keep the crate moving at constant speed?

1. Draw the free-body diagram



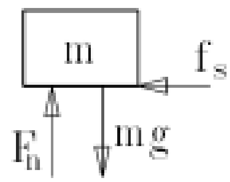
2.Note that $a = 0$. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $F\cos\theta - \mu_k F_n - mg\sin\theta = 0$;

$F_n - F\sin\theta - mg\cos\theta = 0$

3. Solve for F and evaluate $F = \dfrac{mg(\sin\theta + \mu_k\cos\theta)}{\cos\theta - \mu_k\sin\theta}$ 0; $F = 1486$ N

100 ·· Brother Bernard is a very large dog with a taste for tobogganing. Ernie gives him a ride down Idiots' Hill—so named because it is a steep slope that levels out at the bottom for 10 m, and then drops into a river. When they reach the level ground at the bottom, their speed is 40 km/h, and Ernie, sitting in front, starts to dig in his heels to make the toboggan stop. He knows, however, that if he brakes too hard, he will be mashed by Brother Bernard. If the coefficient of static friction between the dog and the toboggan is 0.8, what is the minimum stopping distance that will keep Brother Bernard off Ernie's back?

1. Draw the free-body diagram

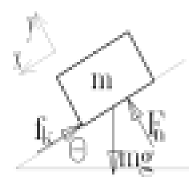


2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ $F_n = mg$; $f_{s,max} = \mu_s F_n = ma_{max}$; $a_{max} = \mu_s g$

3. $s_{min} = v^2/2a_{max} = v^2/2\mu_s g$ $s_{min} = 7.86$ m

101* ·· An object with a mass of 5.5 kg is allowed to slide from rest down an inclined plane. The plane makes an angle of 30° with the horizontal and is 72 m long. The coefficient of kinetic friction between the plane and the object is 0.35. The speed of the object at the bottom of the plane is (*a*) 5.3 m/s. (*b*) 15 m/s. (*c*) 24 m/s. (*d*) 17 m/s. (*e*) 11 m/s.

1. Draw the free-body diagram



2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ $mg\sin\theta - \mu_k F_n = ma$;

$F_n - mg\cos\theta = 0$

3. Solve for a $a = g(\sin\theta - \mu_k\cos\theta)$

4. Use $v^2 = 2as$ $v = \sqrt{2(g\sin\theta - \mu_k g\cos\theta)s} = 16.7$ m/s

5. (*d*) is correct

102 ·· A brick slides down an inclined plank at constant speed when the plank is inclined at an angle θ_0. If the angle is increased to θ_1, the block accelerates down the plank with acceleration a. The coefficient of kinetic friction is the same in both cases. Given θ_0 and θ_1, calculate a.

The free-body diagram is the same as for the preceding problem. We now have $mg\sin\theta_0 = f_k = \mu_k F_n$, and $mg\cos\theta_0 = F_n$. Solving for μ_k we obtain $\mu_k = \tan\theta_0$. With $\theta = \theta_1$, $mg\sin\theta_1 - \mu_k mg\cos\theta_1 = ma$, and using the result $\mu_k = \tan\theta_0$ one finds $a = g(\sin\theta_1 - \tan\theta_0\cos\theta_1)$.

103 ·· One morning, Lou was in a particularly deep and peaceful slumber. Unfortunately, he had spent the night in the back of a dump truck, and Barry, the driver, was keen to go off to work and start dumping things. Rather than risk a ruckus with Lou, Barry simply raised the back of the truck, and when it reached an angle of 30°, Lou slid down the 4-m incline in 2 s, plopped onto a pile of sand, rolled over, and continued to sleep. Calculate the coefficients of static and kinetic friction between Lou and the truck.

1. Use Equ. 5-4 — $\mu_s = \tan 30^\circ = 0.577$
2. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ and use $s = 1/2at^2$ — $F_n = mg\cos\theta$; $mg\sin\theta - \mu_k mg\cos\theta = ma$; $a = 2s/t^2$
3. Solve for and evaluate μ_k — $\mu_k = \tan\theta - 2s/(t^2 g\cos\theta)$; $\mu_k = 0.342$

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104 ·· In a carnival ride, the passenger sits on a seat in a compartment that rotates with constant speed in a vertical circle of radius $r = 5$ m. The heads of the seated passengers always point toward the axis of rotation. (*a*) If the carnival ride completes one full circle in 2 s, find the acceleration of the passenger. (*b*) Find the slowest rate of rotation (in other words, the longest time T to complete one full circle) if the seat belt is to exert no force on the passenger at the top of the ride.

(*a*) $a = r\omega^2 = 4\pi^2 r/T^2$ — $a = 5\pi^2$ m/s^2

(*b*) In this case, $a = a_c = g$; solve for and find T — $T = 2\pi(r/g)^{1/2}$; $T = 4.49$ s

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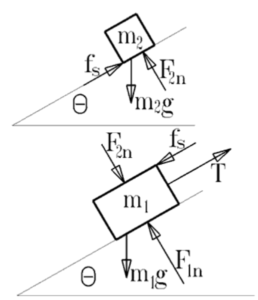
105* ·· A flat-topped toy cart moves on frictionless wheels, pulled by a rope under tension T. The mass of the cart is m_1. A load of mass m_2 rests on top of the cart with a coefficient of static friction μ_s. The cart is pulled up a ramp that is inclined at an angle θ above the horizontal. The rope is parallel to the ramp. What is the maximum tension T that can be applied without making the load slip?

1. Draw the free-body diagrams for the two objects.



1. m_2 is accelerated by f_s. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $F_{n2} = m_2 g\cos\theta$; $\mu_s m_2 g\cos\theta - m_2 g\sin\theta = m_2 a_{max}$
2. Solve for a_{max} — $a_{max} = g(\mu_s\cos\theta - \sin\theta)$
3. The masses move as single unit. Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ — $T - (m_1 + m_2)g\sin\theta = (m_1 + m_2)g(\mu_s\cos\theta - \sin\theta)$
4. Solve for T — $T = (m_1 + m_2)g\mu_s\cos\theta$

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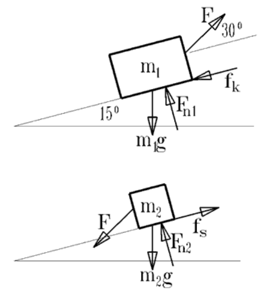
106 ·· A sled weighing 200 N rests on a 15° incline, held in place by static friction (Figure 5-58). The coefficient of static friction is 0.5. (*a*) What is the magnitude of the normal force on the sled? (*b*) What is the magnitude of the static friction on the sled? (*c*) The sled is now pulled up the incline at constant speed by a child. The child weighs 500 N and pulls on the rope with a constant force of 100 N. The rope makes an angle of 30° with the incline and has negligible weight. What is the magnitude of the kinetic friction force on the sled? (*d*) What is the coefficient of kinetic friction between the sled and the incline? (*e*) What is the magnitude of the force exerted on the child by the incline?



Draw the free-body diagrams for the sled (m_1) and the child (m_2).

$F_n = 200\cos 15^o = 193$ N

$f_s = 200\sin 15^o = 51.8$ N

$100\cos 30^o - 200\sin 15^o - f_{s,max} = F_{net}$;

$F_{n1} = 200\cos 15^o - 100\sin 30^o = 143$ N; $f_{s,max} = 0.5 \times F_{n1}$;

$f_{s,max} = 71.5$ N. $F_{net} = -36.7$ N < 0

$F_{n2} = (500\cos 15^o + 100\sin 30^o)$ N $= 533$ N;

$F_x = (500\sin 30^o + 100\cos 30^o)$ N $= 216$ N

$F = 575$ N

(a) $F_n = m_1 g\cos\theta$

(b) Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$

(c) Determine if the sled moves

The sled does not move! f_k is undetermined.

(d) μ_k undetermined.

(e) Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$

Note that the child is stationary

$F = (F_{n2}^2 + F_x^2)^{1/2}$

107 ·· A child slides down a slide inclined at 30° in time t_1. The coefficient of kinetic friction between her and the slide is μ_k. She finds that if she sits on a small cart with frictionless wheels, she slides down the same slide in time $1/2 t_1$. Find μ_k.

1. Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$ for case with and without friction $\quad a_1 = g(\sin 30^o - \mu_k\cos 30^o)$; $a_2 = g\sin 30^o$

2. $s = 1/2 a_1 t_1^2 = 1/2 a_2 t_2^2 = a_2 t_1^2/8$; $a_2/a_1 = 4$; solve for μ_k $\quad \mu_k = (3/4)\tan 30^o = 0.433$

108 ·· The position of a particle of mass m= 0.8 kg as a function of time is $\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} = R\sin\omega t\,\boldsymbol{i} + R\cos\omega t\,\boldsymbol{j}$, where R = 4.0 m, and $\omega = 2\pi$ s^{-1}. (a) Show that this path of the particle is a circle of radius R with its center at the origin. (b) Compute the velocity vector. Show that $v_x/v_y = -y/x$. (c) Compute the acceleration vector and show that it is in the radial direction and has the magnitude v^2/r. (d) Find the magnitude and direction of the net force acting on the particle.

(a) See Problem 5-98. $r = R = 4$ m.

(b) See Problem 5-98. $\mathbf{v} = (\omega R\cos\omega t)\,\mathbf{i} - (\omega R\sin\omega t)\,\mathbf{j} = [(8\pi\cos\omega t)\,\mathbf{i} - (8\pi\sin\omega t)\,\mathbf{j}]$ m/s;
$v_x/v_y = -\cot\omega t = -y/x$.

(c) $\boldsymbol{a} = d\mathbf{v}/dt = [(-16\pi^2\sin\omega t)\,\boldsymbol{i} - (16\pi^2\cos\omega t)\,\boldsymbol{j}]$ m/s^2; note that $\boldsymbol{a} = -4\pi^2\boldsymbol{r}$, i.e., in the radial direction toward origin. The magnitude of $\boldsymbol{a}$ is $16\pi^2$ m/s^2 = $[(8\pi)^2/4]$ m/s^2 = v^2/r.

(d) $F = ma = 12.8\pi^2$ N; the direction of $\boldsymbol{F}$ is that of $\boldsymbol{a}$, i.e., toward the center of the circle.

109* ·· In an amusement-park ride, riders stand with their backs against the wall of a spinning vertical cylinder. The floor falls away and the riders are held up by friction. If the radius of the cylinder is 4 m, find the minimum number of revolutions per minute necessary to prevent the riders from dropping when the coefficient of static friction between a rider and the wall is 0.4.

1. Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$	$F_n = mr\omega^2$; $f_{s,\max} = \mu_s F_n = mg$
2. Solve for and evaluate ω	$\omega = (g/\mu_s r)^{1/2}$; $\omega = 2.476$ rad/s = 23.6 rpm

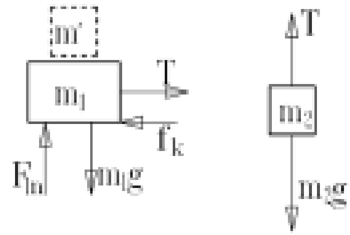
110 ·· Some bootleggers race from the police down a road that has a sharp, level curve with a radius of 30 m. As they go around the curve, the bootleggers squirt oil on the road behind them, reducing the coefficient of static friction from 0.7 to 0.2. When taking this curve, what is the maximum safe speed of (a) the bootleggers' car, and (b) the police car?

(a), (b) Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$	$F_n = mg$; $f_{s,\max} = \mu_s mg = mv_{\max}^2/r$
Solve for $v_{\max}$	$v_{\max} = (\mu_s gr)^{1/2}$
(a) Evaluate $v_{\max}$ for $\mu_s = 0.7$	$v_{\max} = 14.35$ m/s = 51.7 km/h
(b) Evaluate $v_{\max}$ for $\mu_s = 0.2$	$v_{\max} = 7.67$ m/s = 27.6 km/h

111 ·· A mass m_1 on a horizontal shelf is attached by a thin string that passes over a frictionless peg to a 2.5-kg mass m_2 that hangs over the side of the shelf 1.5 m above the ground (Figure 5-59). The system is released from rest at $t = 0$ and the 2.5-kg mass strikes the ground at $t = 0.82$ s. The system is now placed in its initial position and a 1.2-kg mass is placed on top of the block of mass m_1. Released from rest, the 2.5-kg mass now strikes the ground 1.3 seconds later. Determine the mass m_1 and the coefficient of kinetic friction between m_1 and the shelf.

1. Draw the free-body diagrams.
2. Use $s = 1/2at^2$ to find the acceleration a_1 of the first run.
3. Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$ to m_2
4. Evaluate T_1
5. Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$ to m_1.
6. Repeat part 2 for the second run to find a_2.
7. Repeat parts 3 and 4 to find T_2.
8. Apply $\Sigma\boldsymbol{F} = m\boldsymbol{a}$ to m_1 + 1.2 kg.
9. Simplify the preceding result.
10. Solve (1) for μ_k
11. Substitute (3) into (2), simplify to obtain a quadratic equation for m_1.
12. Use the standard solution for m_1.
13. m_1 must be positive, only one solution applies.
7. 14. Substitute m_1 = 1.22 kg into (3) and evaluate

μ_k



$a_1 = (3\text{ m})/(0.82\text{ s})^2$

$a_1 = 4.46\text{ m/s}^2$.

$T_1 - m_2g = -m_2a_1$;

$T_1 = (2.5\text{ kg})[(9.81 - 4.46)\text{ m/s}^2]$; $T_1 = 13.375$ N

$13.375\text{ N} - \mu_k m_1g = m_1(4.46\text{ m/s}^2)$. (1)

$a_2 = (3\text{ m})/1.3\text{ s})^2 = 1.775\text{ m/s}^2$

$T_2 = (2.5\text{ kg})[(9.81 - 1.775)\text{ m/s}^2]$; $T_2 = 20.1$ N

$20.1\text{ N} - \mu_k(m_1 + 1.2\text{ kg})g = (1.775\text{ m/s}^2)(m_1 + 1.2\text{ kg})$

$17.97\text{ N} - \mu_k m_1\text{g} - (1.2\text{ kg})\mu_k g = (1.775\text{ m/s}^2)m_1$ (2)

$\mu_k = [(13.375\text{ N}) - (4.46\text{ m/s}^2)m_1]/m_1g$ (3)

$2.685m_1^2 + 9.947m_1 - 16.05 = 0$

$$m_1 = \frac{-9.947 \pm \sqrt{9.947^2 + 4(2.655)(16.05)}}{2(2.685)}\text{ kg } 0$$

$m_1 = (-1.85 \pm 3.07)$ kg; $m_1 = 1.22$ kg.

$\mu_k = 0.672$

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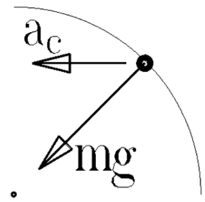
112 ··· (*a*) Show that a point on the surface of the earth at latitude θ has an acceleration relative to a reference frame not rotating with the earth with a magnitude of $3.37 \cos\theta$ cm/s². What is the direction of this acceleration? (*b*) Discuss the effect of this acceleration on the apparent weight of an object near the surface of the earth. (*c*) The free-fall acceleration of an object at sea level measured *relative to the earth's surface* is 9.78 m/s² at the equator and 9.81m/s² at latitude $\theta = 45°$. What are the values of the gravitational field *g* at these points?

(*a*) $R = 6.37 \times 10^8$ cm is the radius of the earth. At a latitude of θ the distance from the surface of the earth to the axis of rotation is $r = R\cos\theta$. The rotational speed of the earth is $\omega = 2\pi/86400\text{ rad/s} = 7.27 \times 10^{-5}$ rad/s. The acceleration is $r\omega^2 = (6.37 \times 10^8 \cos\theta)(7.27 \times 10^{-5})^2\text{ cm/s}^2 = 3.37 \cos\theta\text{ cm/s}^2$. The acceleration is directed toward the axis of rotation.

(*b*) Since the force of gravity supplies the required centripetal force, the acceleration of gravity at the surface of the earth is reduced in magnitude. Consequently, the apparent weight is slightly reduced. This effect is greatest at the equator.

(*c*) The free-body diagram for a mass *m* at $\theta = 45°$ 45°.

is shown. We also show the acceleration at θ =



1. At $\theta = 0^o$, $g_{eff} = g - a_c$
2. At $\theta = 45^o$, $\boldsymbol{g}$ and $\boldsymbol{a}_c$ are not colinear (see diagram). Use law of cosines. Find solution of the quadratic equation.

1. $g = g_{eff} + a_c = (978 + 3.37)\ \mathrm{cm/s^2} = 9.814\ \mathrm{m/s^2}$
2. $g_{eff}^2 = g^2 + a_c^2 - 2ga_c\cos 45^o = g^2 + a_c^2 - 1.41ga_c$

$g^2 - 3.37g - 962350 = 0$

$g = 9.827\ \mathrm{m/s^2}$
